The International Workshop
on Discrete-Event System Design, DESDes'01,
June 27÷29, 2001; Przytok near Zielona Gora, Poland

THE SYSTEM OF HANDWRITTEN CHARACTERS RECOGNITION ON THE BASIS OF LEGENDRE MOMENTS AND NEURAL NETWORK

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Abstract. *This article analyses the image feature extraction task on the basis of Legendre moments for image recognition problem. Computation algorithm of Legendre moments is presented. A new method for training RBF-neural network is introduced. Classification results for binary images (handwritten Arabic numerals) are presented. On the base of classification results the recommendations for choice of maximal order Legendre moments and various classifiers are given.*

Key Words. *Moment invariants, Legendre moments, Neural networks, Radial basis function*

1. INTRODUCTION

Succession of operations in most of digital image recognition systems can be divided into three stages. First stage is a preprocessing, including thresholding, improving image quality, segmentation and so on. Second – features extraction for avoiding data abundance and reducing its dimension. Third stage is a classification. During this stage class name is joint with unknown image by extracted features analyzes and matching its with representatives of the class, which the classifier has learned at a stage of training. In this article two last stages of digital image recognition are presented.

2. LEGENDRE MOMENTS

In feature extraction task considerable attention for methods that use moment functions is given. Moment invariant properties are investigated since sixties [4]. There are invariant on shifts, scaling and rotating of source object. During research time various types of moment functions were introduced, and fast computation algorithms for different types of moments were created. This part of article describes Legendre moments using for handwritten character (Arabic numerals) informative feature extraction.

Two-dimensional Legendre moments for image intensive function *f(x,y)* defines as [6, 8]:

$$L_{kl} = \frac{(2k+1)(2l+1)}{4} \int_{-1}^{1}\int_{-1}^{1} P_k(x) P_l(y) f(x,y) dx dy, \quad (1)$$

where $f(x,y)$ – picture element with coordinates (x,y);

$$P_0(x)=1; \quad P_1(x)=x; \quad P_k(x)=[(2k-1)xP_{k-1}(x)-(k-1)P_{k-2}(x)]/k \text{ –} \quad (2)$$

Legendre polynomial by power k, $k>1$;

$l \geq 0$ и $k \geq 0$ defines the order of moments.

Since definition area of Legendre polynomials is $-1 \leq x \leq 1$, then definition area of two-dimensional Legendre moments is unit square, so a rectangle image of $N \times M$ pixels with intensity function $f(i,j)$, $1 \leq i \leq N$, $1 \leq j \leq M$ will have to be scaled the region $1 \leq x,y \leq 1$, and image center of gravity must be located in the coordinate system origin. For this:

– source image center of gravity (i_c,j_c) is computed;

– distance D from the center of gravity to the farthest from it point of image is determined according to equation

$$\forall (i,j): D = \max\{|i_c - i|, |j_c - j|\}; \quad (3)$$

– scaling of image is performed according to

$$(x,y) = (\frac{j - j_c}{D}, \frac{i_c - i}{D}). \quad (4)$$

Legendre moments to maximum order *MAX_ORDER* can be computed by pseudo-code:

```
for k:=0 to MAX_ORDER
  for l:=0 to k
    L(k-l,l):=0
    for i:=1 to N
      for j:=1 to M
        x:=(j - jc)/D
        y:=(ic - i)/D
        L(k-l,l) := L(k-l,l) + Pk-l(x)*Pl(y)*f(i,j)
      end
    end
    L(k-l,l) := L(k-l,l)*(2k-2l+1)*(2l+1)/(N-1)/(M-1)
  end
end
```

Legendre polynomials $P_k(x)$ forms full orthogonal basis inside unit circle, so source image may be reconstructed from the finite number of Legendre moments as follows:

$$f(x,y) \cong \sum_k \sum_l L_{kl} P_k(x) P_l(y). \quad (5)$$

Fig.1 shows source binary and halftone images and reconstructed images using various number of Legendre moments.

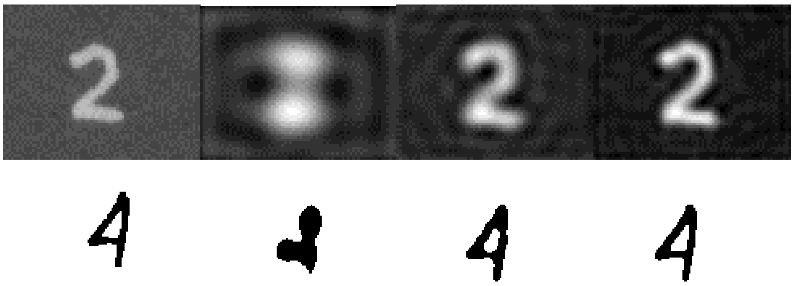

Fig.1. Source and reconstructed images using Legendre moments with maximum order 20, 40 and 60

3. CLASSIFIER ON THE BASIS OF THE RBF-NEURAL NETWORK.

In the tasks of classification the large attention is given to construction of classifiers on the basis of neural networks. Radial basis function (RBF) neural network is two-layer neural network offered by Moody and Darken in 1989 [5] (fig. 2).

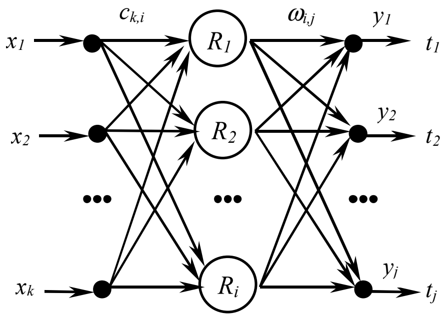

Fig. 2. The RBF- neural network architecture

The RBF-networks represent multilayer neural networks with RBF-neuron layer, which activation function correspond to the radial basic function

$$R(D_i) = \exp\left(-\frac{D_i^{\,2}}{2\cdot\sigma_i^2}\right), \tag{6}$$

were $D_i = |\vec{x} - \vec{c}_i|$ is distance between an entry pattern $\vec{x}$ and i–th cluster of the radial basic function $\vec{c}_i$, σ - width of a cluster.

The RBF-neuron weight coefficients are associated with cluster of the radial basic function. Thus, the output data of RBF-layer represents a vector of closeness measures of entry pattern to all RBF-clusters.

The subsequent layers of such networks usually evaluate a linear combination of these functions.

$$y_j = \sum_i w_{i,j} R_i \,. \tag{7}$$

Key aspect for the RBF-network training is the possibility of layer-by-layer training, that result in two-phase training algorithm: a RBF-layer training and perceptron layer training.

3.1. *RBF-layer training*

The RBF-layer training consists of two tasks: definition of necessary amount of RBF-neurons, and their weight coefficients setting. The training is produced by the following rules:

If in RBF-layer there are no such neurons that $D_i < \sigma$ or amount of neurons is equal to 0, than it is necessary to add a new neuron initializing its weights by training pattern vector value.

Else modification *i*-th neuron weights for which $\min\limits_{\forall i}(D_i)$ is performed

$$\vec{c}_i(t+1) = \vec{c}_i(t) + \frac{1}{t+1} \cdot (\vec{x}(t+1) - \vec{c}_i(t)), \tag{8}$$

were $\vec{x}, \vec{c}_i$ are training vector and cluster vector accordingly, *t*-amount of additions.

3.2. *Perceptron layer training*

The perceptron layer training is made by a gradient descent method with the purpose of the error function minimization in weight coefficient space $w_{i,j}$:

$$Err = \frac{1}{2} \cdot \sum_n \sum_j \left(\sum_i w_{i,j} \cdot R_i^n - t_j^n \right)^2 \to 0, \tag{9}$$

were *n=[1..N]* is amount of learning images, t_j^n - target value of an *j-th* output for a training pattern *n*.

4. **CLASSIFICATION RESULTS**

A part of binary image database used in experiments is presented in Fig. 3. Database consists of 10 classes of images (10 Arabic numerals). Each class consists of 225 objects. 175 objects from each class are used for training and 50 – for recognition.

For classification the RBF neural network classifier and minimal distance classifier [2] were used. Classification results for different maximum order of Legendre moments is presented in Table 1.

Table 1. Binary images classification results

MAX_ORDER	***Recognition percent, %***	
	Minimal distance classifier	***Neural network***
5	76,4	92
10	90,6	98,2
15	91,6	98,7
20	88,4	98,7
25	85,2	98,5

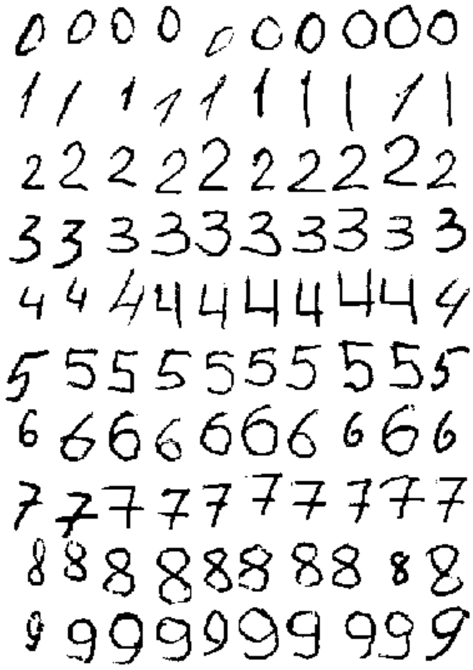

Fig3. Part of binary images database

5. CONCLUSION

In this paper the problem of handwritten characters (Arabic numerals) recognition was examined. Legendre moments properties as a classifiers features were investigated. Also characteristics of new classifier based on neural network in comparison with minimal distance classifier were tested. On classification results the next conclusions can be done:

- during classification performing it is no purpose to use moment functions with maximum order above 20.

- classification must be performed using neural network classifiers.

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